



## THEORETICAL AND EXPERIMENTAL STUDY OF COUNTER FLOW WET COOLING TOWERS

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### ABSTRACT:

This paper presents an accurate analysis for steady state counter flow wet cooling tower. This analysis considers the water loss by evaporation and water/air interface thermal resistance, so it considers the actual driving force for cooling process. The present model considers the non-unity of Lewis number and the curvature for the saturated air enthalpy curve where a polynomial of tenth degree was derived for temperature ranged from 10 to 60 °C . A modified expression for the number of transfer units (NTU) based on the minimum capacity rate of air and water was applied. Also, the paper describes the effectiveness-NTU design method for cooling towers. Experimental runs were carried out on a small-scale cooling tower to verify the present model and to determine the test tower fills constants. Also, the present tower characteristic equations are obtained. The present analysis showed that the water evaporation and the thermal resistance of water/air interface film lead to oversize the cooling tower by 300% under bad conditions of heat transfer.

### ملخص البحث

يقدم هذا البحث تحليلاً دقيقاً لبرج تبريد رطب لتبريد الماء بالهواء يعمل تحت ظروف مستقرة ويكون اتجاه سريان الماء عكس اتجاه سريان الهواء. يأخذ هذا التحليل في الاعتبار كمية المياه المفقودة في البخر وأيضاً المقاومة الحرارية للطبقة الرقيقة الفاصلة بين الماء والهواء؛ لذا فهو يأخذ في الاعتبار القوة الحقيقية المحركة والمسببة لانتقال كل من الحرارة والكتلة.

والنموذج الحالي يأخذ في الاعتبار ظروف تشغيل لرقم لويس مختلفاً عن الواحد الصحيح وأيضاً يأخذ في الاعتبار انحناء خط التشبع للهواء (درجة الحرارة-الانتالي) وذلك باشتقاق معادلة كثيرة حدود من الدرجة العاشرة في مدى درجات حرارة يتراوح من 10 إلى 60 درجة مئوية، كما يصف استخدام الطريقة المعروفة باسم (الفاعلية-عدد وحدات الانتقال) في تصميم برج التبريد كما تم تطبيق تعريف لعدد وحدات انتقال كل من الحرارة والكتلة يعتمد على أقل معدل سعة للماء والهواء. وتم إجراء تجارب معملية على نموذج مصغر لبرج تبريد لتطبيق التحليل الرياضي الحالي عليه وأيضاً لتحديد ثوابت مادة الحشو المستخدم في برج التبريد؛ وقد تم الحصول على المعادلة المميزة للبرج المستخدم وبين البحث أن المقاومة الحرارية للطبقة الرقيقة الفاصلة بين الماء والهواء وأيضاً كمية الماء المتبخر

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قد يؤديان إلى زياده في حجم البرج تصل إلى ٣٠٠% من حجمه في حالة إهمالهما وذلك تحت أسوأ ظروف لانتقال الحرارة.

## 1- INTRODUCTION

Cooling towers are used to reduce the temperature of the circulating water so that it may be reused in condensers and other heat exchange equipment. This process is accomplished by bringing the hot water into direct contact with the unsaturated air. This method is only applicable in cases where the wet bulb temperature of the ambient air is less than the required minimum temperature of the circulating water. Cooling towers are most efficient devices to achieve this task. Another strong motivation for the increased use of the cooling towers is the environmental protection provided through the reduction of water withdrawals and minimum thermal discharge. Cooling towers are used in many applications such as power generation units, air conditioning, petrochemical and petroleum industries.

The advantages of cooling towers include the absence of heat transfer surface which represent a thermal resistance and can be fouled or corroded. So, higher rates of heat and mass transfer and lower initial and operating costs are obtained. Water cooling towers are sized and selected based on economic consideration and constraints imposed by system components. However, accurate analysis of cooling towers must be available to ensure a precise cooling water temperature. This temperature is of major importance which has direct contact on the economics of the design.

The theoretical analysis of wet cooling towers has a long history which has led to a huge number of publications. The basic theory of cooling tower operation was first proposed in 1923 by Walker et al [1]. However, the first practical use of the differential equations was developed by Merkel in 1925 [2]. In his work, the equations for heat and mass transfer were used, and the enthalpy was used as the driving force to allow for both sensible and latent heat transfer. Heat is removed from water by a transfer of both sensible heat due to a difference in temperature levels and the latent heat due to evaporation of a portion of the circulating water. Merkel combined these into a single process based on enthalpy difference as the driving force. The theory used by Merkel requires two main assumptions;

- 1- the water loss by evaporation is neglected.
- 2- the Lewis number for air/water vapor system is unity. Different authors have used different definitions for the Lewis number, Le, Webb [3] compared all these definitions and concluded that the ratio of thermal diffusivity to the mass diffusivity is the most meaningful one.

The theory states that all of heat transfer taking place at any position in the tower is proportional to the difference between the enthalpy of air saturated at the temperature of the water and the enthalpy of the air at that position in the tower. Quantitative treatment of cooling tower performance by dealing with heat and

mass transfer separately is very laborious. Therefore, the simplifying approximation of Merkel's enthalpy theory has been almost universally adopted for the calculation of tower performance.

Merkel's differential equation for the cooling tower was redeveloped by Nottage [4] and converted to a graphical method of solution by Lichtenstein [5] and another graphical procedure for determining the air condition line by Mickley [6]. Extensive sets of curves for cooling tower design, based on Merkel's theory, have been proposed by ASHRAE, [7].

Experimental studies on several small-scale cooling towers, are carried out by Simpson and Sherwood [8]. They examined the dependence of the mass transfer coefficient on the various air and water properties. Carey and Williamson [9] extended Merkel's theory to be applicable to gas cooling and humidification, and proposed the Stevens diagram for the solution of cooling tower characteristic necessary for determining the required volume of a tower.

One source of errors in the cooling tower analysis is due to water loss by evaporation. Sutherland [10] developed a computer program to compare the accurate analysis of mechanical draught counter flow cooling towers, including water loss by evaporation, with the approximate Merkel's method. The results showed that a substantial underestimates of tower volume, of range from 5 to 15% when approximate analysis is used. Nahavandi et al [11], showed that ignoring the evaporation losses introduces an error in the Merkel results which is not conservatives and may reach 12% depending on the design conditions. Also the effect of water evaporation on the cooling tower performance has also been studied by Baker[12], Threlkeld[13], Webb[14], and Yadigaroglie and Poster [15].

Another Source of errors which has been examined is due to the resistance to heat transfer in the water film and the non-unity values of Lewis number. Raghavan [16], Stevens et al [17], and Jefferson [18], introduced an adjustment coefficient to account for the effect of the actual values of the Lewis number.

Majumdar et al [19] reevaluated the thermal performance of the cooling towers through the introduction of mathematical model for mechanical and natural draft cooling towers. The model is based on two dimensional analysis and computed the air velocity, temperature, pressure and moisture content and the water temperature. Sadasivam and Balakrishnan [20], introduced a new definition for driving potential available for the net heat transfer in counter current cooling towers based on the apparent enthalpy. This apparent enthalpy is not limited by Lewis relation, i.e. Lewis number need not necessarily to be one.

El-Dessouky et al [21], introduced a theoretical investigation for the steady state counter flow wet cooling tower with modified definitions for both the number of transfer units and the tower thermal effectiveness. A new expression relating the tower effectiveness to the modified number of transfer units and the capacity rate ratio has been developed. Their model uses non-unity Lewis number. The model

was compared very satisfactory with other methods such as Logarithmic Mean Enthalpy Difference and conventional effectiveness-NTU.

The objectives of the present work is to introduce an efficient analysis for counter flow wet cooling towers to overcome the limitations that exist in the previous analyses and study the effect of different parameters on the cooling tower operation and design. Also, several experiments were carried out on small-scale cooling tower\* to validate the present analysis.

## 2- EXPERIMENTAL APPARATUS

The present cooling tower\* test rig is an open system through two streams of fluid flow (water and air) and which there is a mass and heat transfer from one stream to the other. It has the same configuration as a full size forced draught cooling tower. The tower column is made of clear PVC and has a dimension of 150 x150x600 mm high. The tower contains eight decks and inclined to the horizontal by an angle of 60°. Each deck contain ten wettable, laminated plastic plates, retained by water distribution troughs. The total surface area to volume ratio 110 m<sup>2</sup>/m<sup>3</sup>. The components of the tower are shown in Fig. (1) This figure includes air distribution chamber, a tank with heaters to simulate heating loads of 0.5 , 1.0 and 1.5 kW. There are also a make up tank with gauge mark, floating operated central valve, and centrifugal fan with intake damper to give a maximum air flow of 0.06 kg/s, bronze and stainless steel glandless circulating pump driven by 100 Watt motor and a water collecting basin. Warm water is pumped from the load tank through the control valve and the flowmeter to the column cap. The water is uniformly distributed over the top packing deck. Air from the atmosphere enter the tower by the fan at a rate which is controlled by the intake damper setting. Flow through the column may be observed through the transparent PVC casing. Six point digital temperature indicator with K-type thermocouple sensors is used to measure terminal water temperatures, wet and dry bulb air temperature. Pressure tappings are also provided with inclined manometer of range (0-40 mm of H<sub>2</sub>O) is used to measure for air flow rate and the packing resistance. Also the water mass flow rate which enters the tower is controlled by a valve and measured through a flowmeter of range (0-50 gm/s).

## 3- MATHEMATICAL ANALYSIS AND METHOD OF SOLUTION

### 3-1 Mathematical Analysis

The analysis presented here satisfies the following assumptions:

- i.- The cooling tower is of wet counter flow type and operates under steady-state conditions
- ii- Thermal loads such as make-up water additions, pump heat gain and net exchange with the surrounding are negligible.

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- iii-Thermodynamic properties of the upward air flow and downward water flow vary vertically
- iv-The specific heats at constant pressure of the two fluids are constant throughout the cooling tower.
- v- The heat and mass transfer coefficients are constant in the packing region.

Consider the direct contact counter flow cooling tower over the differential volume,  $dV$ ,  $A'B'C'D'$ , as shown in Fig. (2). The mass balance of water over the element,  $dV$ , gives rise to the following equation:

$$dM_w = m_a dw \quad (1)$$

The heat balance over the element,  $dV$ , gives:

$$m_a dH_a = - [M_{w1} - m_a (w_2 - w)] dH_{fw} + m_a H_{fw} dw \quad (2)$$

The rate of evaporation of water on the element,  $dV$ , can be expressed as:

$$m_a dw = h_m A_v dV (w_{sw} - w) \quad (3)$$

Spalding [22] used the mass fraction  $w/(1+w)$  as the driving force for mass transfer which can be shown to be equivalent to Eq. (3). This equation is also used by Baker and Shryock [23], Threlkeld [13], and Maclaine-cross and Bank [24].

The heat lost by water through convection and evaporation, gives the following equation:

$$m_a dH_a = h_a A_v dV (T_w - t_{db}) + h_m A_v dV (w_{sw} - w) H_{gw} \quad (4)$$

where;

$$H_{gw} = H_{fw} + H_{fgw}$$

The Lewis relationship is necessary to combine the transfer of sensible heat into an overall coefficient based on enthalpy difference as the driving force. Using Lewis number,  $Le$ , definition as:

$$Le = \frac{h_a}{h_m C_{ma}}; \quad (5)$$

and dividing Eq. (4) by Eq. (3) gives:

$$\frac{dH_a}{dw} = \frac{Le C_{ma} (T_w - t_{db})}{w_{sw} - w} + H_{gw} \quad (6)$$

Defining the enthalpy of the moist air as:

$$H_a = C_a t_{db} + [C_v t_{db} + H_{go}] w$$

$$H_a = C_{ma} t_{db} + H_{go} w = (H_{sw} - H_a) - H_{gw} (w_{sw} - w) \quad (7)$$

substituting from Eq. (7) into Eq. (6), so,

$$\frac{dH_a}{dw} = Le \left( \frac{H_{sw} - H_a}{w_{sw} - w} \right) + H_{gw} (1 - Le) \quad (8)$$

The solution of the above differential equation determines the enthalpy of the air for a certain independent value of specific humidity. Also, the temperature of water can be determined by dividing Eq. (3) by  $m_a dw$  and substituting

$$dH_{fw} = C_w dT_w.$$

$$\frac{dT_w}{dw} = \frac{\frac{dH_a}{dw} - H_{fw}}{C_w \left[ \frac{M_{wl}}{m_a} - (w_2 - w) \right]} \quad (9)$$

The above analysis is similar to that of Sutherland [10] and is modified by introducing the driving potential as presented by Webb [25];

$$dq = h_a Le^{2/3} (T_i - t_{db}) + H_{gi} h_m (w_i - w) dA \quad (10)$$

where the first term accounts for the single phase heat transfer from water-air interface to the water evaporated at the interface. Webb [25], showed that above equation can be approximated as;

$$dq = h_m (H_{si} - H_a) dA \quad (11)$$

The definition of the parameters used in the present cooling water analysis is illustrated in Fig. (3). A plot of air enthalpy versus water temperature for a counter flow cooling tower is shown in Fig (4). The curved line is the enthalpy of saturated air  $H_s$  and the straight line is the air operating line. The driving force  $(H_{si} - H_a)$  is illustrated by the dashed lines. The previous mathematical analysis assumes that the water-air-interface temperature is equal to the local mixed water temperature i.e.  $h_w/h_m = \infty$ , which is an approximation. Actually the interface temperature is less than the local mixed water temperature. The slope of tie line  $-(h_w/h_m)$  define the interface temperature. The number of the transfer units NTU, before considering the effect of the interface resistance is given by  $\int \frac{dw}{w_{sw} - w}$ , will be changed as

given by Jaber and Webb [26] as follows:

$$NTU = \int \frac{d(H_{si} - H_a)}{H_{si} - H_a} = \frac{h_m A}{m_{min}} \quad (12)$$

where;  $m_{min}$  is the minimum value of either  $m_w^+$  or  $m_a$  and  $m_w^+$  is given by:

$$m_w^+ = \frac{M_w C_w}{f}, \quad f = \frac{dH_{si}}{dT_w} \quad (13)$$

where  $H_s = f(T_w)$  is derived in this work by a polynomial of 10<sup>th</sup> order for a temperature range of (10-60 °C). The polynomial constants are as follows:-

$$H_s = a_0 + \sum_{n=1}^{10} a_n t^n \quad (14)$$

$a_0 = 8.36537$ ,  $a_1 = 2.0482$ ,  $a_2 = -0.016547$ ,  $a_3 = 0.202296$ ,  $a_4 = 2.0848 \times 10^{-5}$ ,  
 $a_5 = -5.7473 \times 10^{-6}$ ,  $a_6 = 2.9383 \times 10^{-7}$ ,  $a_7 = -7.585238 \times 10^{-9}$ ,  $a_8 = 1.1162486 \times 10^{-10}$ ,  
 $a_9 = -8.8796 \times 10^{-13}$ , and  $a_{10} = 3.00468 \times 10^{-15}$ , where (t) is substituted in °C

It is necessary to define heat exchange effectiveness ( $\epsilon$ ). It will be defined identically to that used for heat exchanger design;

$$\epsilon = q_{act}/q_{max} \quad (15)$$

$$\text{where; } q_{max} = m_{min} (H_{si} - H_{a1}) \quad (16)$$

The  $\epsilon$ -NTU equation for counter flow cooling tower is given by:

$$\epsilon = \frac{1 - \exp[-NTU(1 - C_R)]}{1 - C_R \exp[-NTU(1 - C_R)]} \quad (17)$$

The properties of moist air and water are required for solving the differential equations (8) and (9) and the complete cooling tower analysis. The saturated vapor pressure of water ( $P_s$ ) is calculated from the following equation, [27].

$$\ln(P_s) = C_1/T + C_2 + C_3 T + C_4 T^2 + C_5 T^3 + C_6 \ln T \quad (18)$$

where;  $C_1 = -5800.2206$ ,  $C_2 = 1.3914993$ ,  $C_3 = -0.04860239$ ,  $C_4 = 0.41764768 \times 10^{-4}$ ,  $C_5 = -0.14452093 \times 10^{-7}$  and  $C_6 = 6.5459673$  and  $T$  in K.

The actual vapor pressure in terms of the wet bulb temperature is calculated from Regnault/August/Apihjon equation [28]:

$$P_v = P_{s,wb} - 6.67 \times 10^{-5} P_{bar} (t_{db} - t_{wb}) \quad (19)$$

The specific volume of the water vapor is given by:

$$v = \frac{R T}{M_w P_v} \quad (20)$$

$$m_a = \frac{P_a V}{R_a T_a} \quad (21)$$

The specific humidity is given by:

$$w = \frac{m_v}{m_a} \quad (22)$$

### 3.2 Method of Solution

In the present analysis two models are used for solving the counter flow wet cooling tower, the first model deals with the above mathematical analysis and the second model deals with the simplifications used in Merkel's theory ( $Le=1.0$  and  $dM_w = 0.0$ ). The solution of the simplified analysis is carried out as mentioned in [7]. While the governing equations for air and water represented by Eqs. (8) and (9) are solved by a fourth order Runge-Kutta method [29]. The initial values at the bottom of the tower  $w_1, H_{a1}, T_{w2}$  must be given. Thus for increasing the moisture content  $w$ , by small steps, the values of  $H_a$  and  $T_w$  can be calculated.

### 4- MODELS VALIDATION

To check on the accuracy and reliability of the present models, some comparisons were made with published values of cooling tower integrals (number of transfer units, NTU), as shown in Table 1

Table 1: Comparison of results from the present model with literature

$M_{w1}$	$T_{w1}$	$T_{w2}$	$t_{db1}$	$t_{wb1}$	NTU accurate analysis			NTU Merkel theory		
					Thre-keld [13]	Suther-land [10]	present	Macla-ine et al [24]	Suther-land [10]	present
1.2	32.22	26.67	26.67	21.11				1.053	1.054	1.0535
1.2	48.89	26.67	26.67	21.11				2.878	2.886	3.0126
1.0	37.78	29.44	35.00	23.89	1.162	1.188	1.186			

#### Examples of Sizing and Rating Calculations

The sizing problem could be solved by determining NTU (size of the cooling tower) for given air and water conditions. The rating calculations

determine the leaving water temperature for given air and water inlet conditions and tower volume (characteristic). The purpose of these examples is to check the validity of the present model through the comparison of its results with those available in the literature.

### **Sizing Calculations**

Consider the same conditions given by Jaber and Webb [26]:

Water inlet temperature = 35 °C ; Water outlet temperature = 30 °C

Air inlet wet bulb temperature = 25 °C ;  $M_w/m_a = 1.0$

So, some results of the present model and Jaber and Webb are obtained as follows:-

	Jaber and Webb [26]	Present
Air enthalpy change, kJ/kg	20.93	21.0
Air exit enthalpy, kJ/kg	97.53	97.45
Slope of saturation line kJ/kg K	5.916	5.89
Tower effectiveness $\epsilon$	0.555	0.563
Heat capacity rate ratio	0.708	0.710
NTU calculated as $\frac{h_m A}{M_w}$	1.051	1.070
NTU calculated as $\frac{h_m A}{m_{min}}$	0.740	0.760

### **Rating Calculations**

Assume that the given tower has the following data;

$h_m A/m_w = 1.87$ ,  $M_w/m_a = 1.0$  ; water inlet temperature = 35 °C

air inlet wet bulb temperature = 20 °C

It is required to evaluate the exit water temperature, (the actual exit water temperature is 25 °C). For sake of illustration the exit water temperature is really unknown, so, it is assumed to be 29 °C. The following steps are carried out as in the sizing problem

	Jaber and Webb[26]	Present
Enthalpy correction factor $\delta$ , kJ/kg	0.623	0.60
Air exit enthalpy, kJ/kg	97.53	97.45
Slope of saturation line kJ/kg K	5.78	5.77
Tower effectiveness $\epsilon$	0.486	0.48
Heat capacity rate ratio	0.724	0.728
NTU calculated as $\frac{h_m A}{M_w}$	0.608	0.61
NTU calculated as $\frac{h_m A}{m_{min}}$	0.84	0.84

One obtains NTU=0.84, as compared to the given value of 1.87. One may continue the iteration calculations until the calculated tower characteristics agree with the given value.



## 5- RESULTS AND DISCUSSIONS

There are two groups of results, the first one is the numerical results obtained from the present model and the other is the experimental one which is obtained from the test rig.

### 5.1 Numerical Results

#### 5.1.1 Effect of water/air mass ratio

It is now necessary to show the results of the model taking into account the water evaporation and water/air interface thermal resistance. The following table illustrates the operating conditions for five runs. The last two columns in the table refer to the approach to equilibrium ( $A_p$ ) which is defined by the temperature difference between the water outlet temperature and the inlet air wet bulb temperature and the range of cooling (Range) which is the temperature difference between inlet and outlet water temperatures. These runs were used for water/air mass ratio ranges from 0.5 to 1.5, keeping  $m_a = 10$  kg/s throughout. The standard value of the atmospheric pressure 101.3 kPa was used. The convective Lewis number  $Le$  was set at 1.0 and a typical value of  $h_m A_v = 0.5$  ( $\text{kg/m}^2\text{s})(\text{m}^2/\text{m}^3)$  as stated by Sutherland [10], Threkled [13]. The value of tie line slope as stated from the literature ranges from 15-45 ( $\text{kW/m}^2\text{K})/(\text{kg/m}^2\text{s})$ . This tie line shows that the air/water interface film temperature is less than the water temperature, so it considers the film thermal resistance and the actual temperature difference between the film and adjacent air layer. In these five runs, the slope of tie line  $-h_w/h_m$  was having a value of 15 ( $\text{kW/m}^2\text{K})/(\text{kg/m}^2\text{s})$  as supported by Webb [25].

Table 2 : Initial Operating Conditions for Runs of the Model

Run	$T_{w1}$ °C	$T_{w2}$ °C	$t_{wh1}$ °C	$t_{dbl}$ °C	$A_p = T_{w2} - t_{wbl}$ °C	Range = $T_{w2} - T_{w1}$ °C
1	50	40	30	35	10	10
2	50	40	20	25	20	10
3	40	30	20	25	10	10
4	50	40	10	15	30	10
5	40	30	10	15	20	10

Figure (5a) shows the ideal number of transfer units (tower characteristic or integral) obtained from the approximate analysis illustrated by Merkel. A little effect of water/air mass ratio on the number of transfer units for runs 1,2,4 and 5 while for run 3 more dependence of the ideal number of transfer units on the water/air mass ratio is noticed.

The effect of water evaporation on the number of transfer units is illustrated in Fig. (5b). More dependence of number of transfer units on the water/air mass ratio is noticed and this dependence increases as the mass ratio increases. The

variation of transfer units with the mass ratio, taking into account the combined effects of water evaporation and water/air interface thermal resistance with  $h_w/h_m=15$  (kW/m<sup>2</sup>K)/(kg/m<sup>2</sup>s), is depicted in Fig. (5c). From these figures for a fixed value of  $m_w/m_a$ , the following remarks are noticed:-

- 1- For the same range and approach, the higher the inlet wet bulb temperature the smaller the number of transfer units and consequently smaller tower volume,  $NTU_1 < NTU_3, NTU_2 < NTU_5$ .
- 2- For the same wet bulb temperature and range the larger the approach the smaller is the tower  $NTU_2 < NTU_3, NTU_4 < NTU_5$ .
- 3- For the same initial and final water temperatures, the lower the inlet air wet bulb temperature the smaller is the tower  $NTU_4 < NTU_2 < NTU_1, NTU_5 < NTU_3$ .

Referring to Fig. (6), due to neglecting the effect of water evaporated in the tower analysis the percentage error in tower volume at  $m_w/m_a=0.5$  ranges from about 5-10%, (average 7.5%), and for  $m_w/m_a=1.5$  the percentage error ranges from 7-12%, (average 8.5%). This means that as the water/air mass ratio increases the percentage error in tower volume increases. This percentage error is based on the ideal (Merkel) volume.

Figure (7) shows the variation of the percentage error in tower volume with water/air mass flow rate ratio when considering both the effect of water evaporation,  $h_m A_v = 0.5$  (kg/m<sup>2</sup>s)(m<sup>2</sup>/m<sup>3</sup>), and water/air interface film thermal resistance defined by  $-h_w/h_m=15$  (kW/m<sup>2</sup>K)/(kg/m<sup>2</sup>s), higher thermal resistance. From the figure, it can be noticed that  $m_w/m_a=0.5$  the percentage error in volume based on Merkel's volume has an average value of 300% and for  $m_w/m_a=1.5$  has an average value of 100%. This indicates that the percentage error in tower volume is decreasing with increasing water/air mass flow rate ratio. Also, the thermal interface resistance has great effect on the tower volume and the value of  $h_w/h_m$  must be as maximum as possible. Also, From the figure, it can be recommended that for lowest heat transfer rate, the mass flow rate ratio must be maximum.

### 5.1.2 Effect of inlet air conditions

Numerical results were obtained for different inlet conditions for counter flow cooling tower using modified  $\epsilon$ -NTU method described in the above analysis. The height of the column required to accomplish a prescribed cooling duty was determined for Lewis number having values of 0.8, 0.9 and 1.0. The tower parameters and inlet air conditions are given in Table 3.

Table 3: Operating Conditions for Runs of the Model

$t_{db}$	27	29	31	33	35
$t_{wb}$	25.7	26.2	26.6	27	27.5
$A_v=42 \text{ m}^2/\text{m}^3$ , $h_m=0.054 \text{ kg}/\text{m}^2\text{s}$ , $h_w/h_m=45 \text{ (kW}/\text{m}^2\text{K})/(\text{kg}/\text{m}^2\text{s})$ , $G=L=1 \text{ kg}/\text{s m}^2$ , $T_{w, out} = 30 \text{ }^\circ\text{C}$ , $T_{w, in} = 35 \text{ }^\circ\text{C} \text{ \& } 40 \text{ }^\circ\text{C}$					

Figure (8a) presents the effect of air inlet temperature on the percentage error in tower height obtained by the present model taking into account water evaporation only for inlet water temperature 35 °C. It can be concluded that from the figure that as Lewis number increases the percentage error in tower height increases and for all Lewis number different values the curves meet at a particular point then the trend is reversed. This indicates that at this point Lewis number does not have any significant effect. Also, from the figure, it can be seen that the percentage error ranges for 4-1%. The same behavior takes place for inlet water temperature 40 °C and error ranges from about 8 - 13%.

Due to water/air interface film thermal resistance assumption being equal to zero and also neglecting the mass of water evaporation throughout the tower the required tower height would be greater than that of ideal Merkel's height by a percentage of about 55% for inlet water temperature 35 °C and about 73.5% for inlet water temperature 40 °C.

## 5.2 Experimental Results

Several experimental runs were carried out on the test rig shown in Fig. (1). The present theoretical analysis is checked for the consistency and reliability by comparing the computed volume of the cooling tower from Merkel's approximate analysis and the present accurate analysis with the true volume of the tower as shown in Fig. (10). From the figure it is shown that, for water/air mass ratio up to 0.6 Merkel's analysis gives undersize whereas the accurate analysis gives oversize. For water/air mass ratio greater than 0.7, both the accurate and Merkel analyses give the value of the tower. Through the experimental runs which carried out on the test bench, the empirical mass transfer coefficient,  $h_m$ , is determined and the existing tower characteristic is calculated and this estimated value from the model is compared with the actual tower volume to verify the model results. For counter flow cooling tower Lowe and Christie [30] have reported empirical data in the following form:

$$\frac{h_m A_v}{L} = \lambda \left( \frac{M_w}{m_a} \right)^{-n} \quad (23)$$

where;  $\lambda$  and  $n$  are empirical constants. Different fills have different values of  $\lambda$  and  $n$ .

Figure (11) shows the data for inclined rectangular plastic fills used in the present tower. The data is used to correlated the mass transfer coefficient with water/air mass flow rate ratio with  $\lambda = 2.773$  and  $n = 0.523$  with percentage error of 8.6 %. The variation of the present tower characteristic based on the water mass flow rate with water/air mass ratio is illustrated in Fig. (12). The data in the figure is used to get the following correlation;

$$\frac{h_m A}{M_w} = 1.3413 \left( \frac{M_w}{m_a} \right)^{0.4678} \quad (24)$$

with percentage error of 1.48%. If the tower characteristic is calculated based on the minimum mass flow rate value of  $m_w^* = \frac{M_w C_{p_w}}{f}$  or  $m_a$ . The variation of this tower characteristic is shown in Fig. (13) and the data in the figure is correlated as:

$$\frac{h_m A}{m_{\min}} = 1.1827 \left( \frac{M_w}{m_a} \right)^{-0.45898} \quad (25)$$

with percentage error of 4 %.

## 6- CONCLUSIONS

From the present analysis and results, the following conclusions can be drawn:

- 1- Counter flow cooling tower is significantly undersized if the approximate analysis by Merkel is employed.
- 2- The effect of water evaporation in the cooling process is considerable especially for higher water/air mass flow rate ratio  $M_w/m_a=1.5$  reaches to 12% oversized than Merkel size..
- 3- The thermal resistance of water/air interface film has great effect on the tower size especially for lower water/air mass flow ratio  $M_w/m_a=0.5$ ,  $h_w/h_m=15$  (kW/m<sup>2</sup>K)/(kg/m<sup>2</sup>s) reaches to 300% oversized than Merkel size.
- 4- The analysis presented here shows how the  $\epsilon$ -NTU method of heat exchanger design may be applied to the cooling tower with simple definitions of NTU and  $\epsilon$ .
- 5- The empirical constants of the plastic fills used in the present tower is obtained. Also, the average mass transfer coefficient  $h_m$ , for this tower is calculated and has an average value of mass transfer coefficient  $h_m=0.054$  (kg/m<sup>2</sup> s) over the range of experiments carried out in the present work.
- 6- The characteristic equations of the present tower is obtained.

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## NOTATIONS

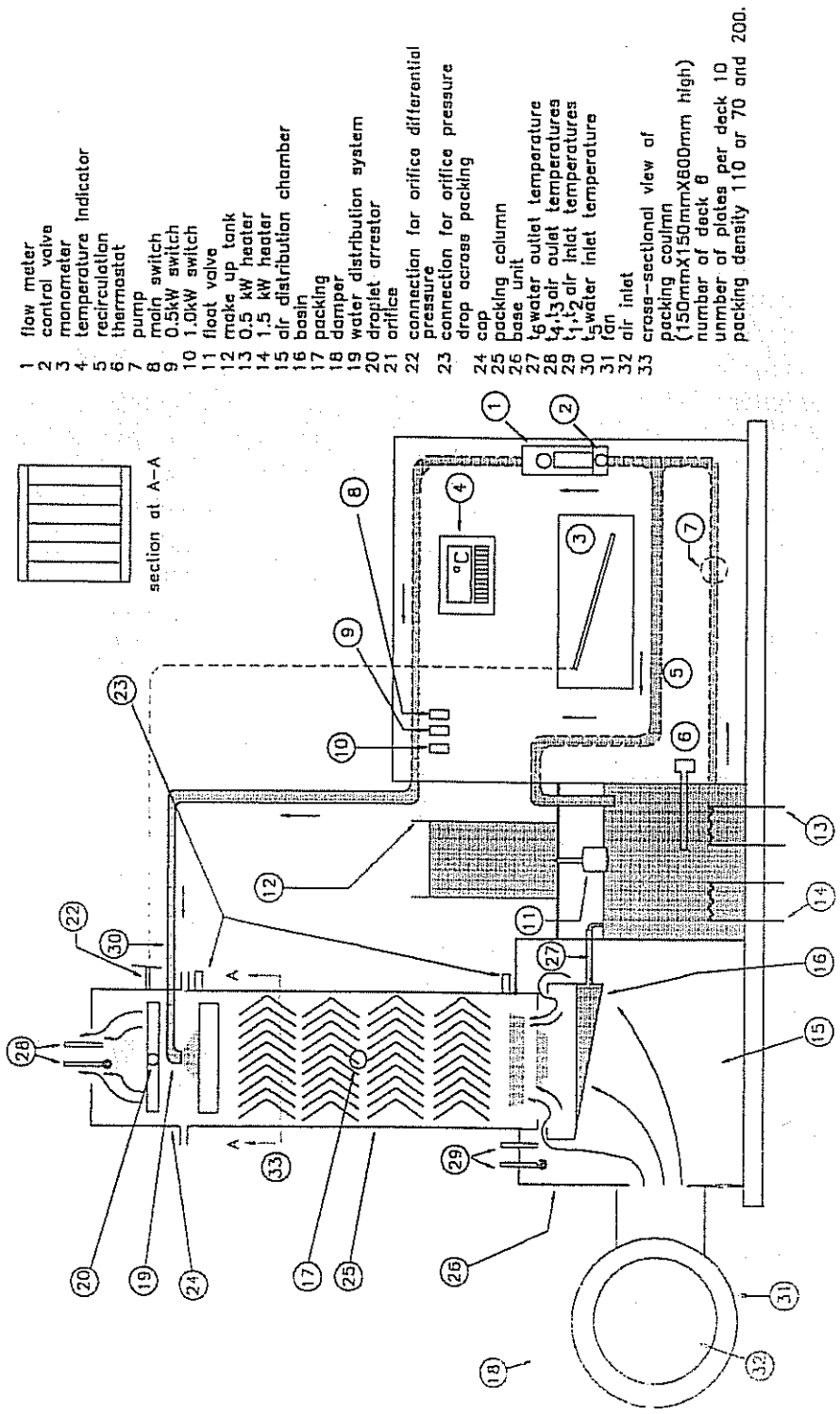
$A$	area, $m^2$	$h_w$	convective heat transfer coefficient, at water side, $kW/m^2K$
$A_p$	approach, $^{\circ}C$	$h_m$	mass transfer coefficient, $kg/m^2 s$
$A_v$	area to volume ratio $m^2/m^3$	$Le$	Lewis number
$a_0, \dots, a_{10}$	coefficient in Eq. (14)	$m_a$	air mass flow rate, $kg/s$
$C_a$	specific heat at constant pressure for air, $kJ/kg K$	$M_w$	water mass flow rate, $kg/s$
$C_{ma}$	specific heat at constant pressure for moist air, $kJ/kg K$	$m_v$	water vapor mass flow rate, $kg/s$
$C_v$	specific heat at constant pressure for water vapor, $kJ/kg K$	$m_w^+$	water side capacity rate, $kg/s$
$C_w$	specific heat for water, $kJ/kg K$	NTU	number of transfer units
$C_1, \dots, C_6$	coefficient in Eq. (18)	$P_a$	dry air partial pressure, $kPa$
$C_R$	heat capacity rate ratio; $m_{min}/m_{max}$	$P_v$	water vapor partial pressure, $kPa$
$f$	saturated air enthalpy function, $kJ/kg$	$q$	heat transfer rate, $kW$
$f'$	slope of saturated enthalpy vs. temperature, $kJ/^{\circ}C$	$R$	universal gas constant, $kJ/kmol K$
$H_a$	enthalpy of moist air $kJ/kg$	$T_w$	water temperature, $^{\circ}C$
$H_{fw}$	enthalpy of liquid water at $T_w$ , $kJ/kg$	$T_i$	air-water interface temperature, $^{\circ}C$
$H_{gw}$	enthalpy of water vapor at $T_w$ , $kJ/kg$	$t_{db}, t_{wb}$	air dry and wet bulb temperature,
$H_{si}$	enthalpy of saturated air at water/air interface temperature, $kJ/kg$	$V$	cooling tower volume, $m^3$
$H_{go}$	enthalpy of water vapor at $0^{\circ}C$ , $kJ/kg$	$v$	air specific volume, $m^3/kg$
$H_{sw}$	enthalpy of saturated moist air at $T_w$ , $kJ/kg$	$w$	humidity ratio, $kg_{gw}/kg_{d.a}$
$H_{gi}$	enthalpy of saturated water vapor at air-water interface temperature, $kJ/kg$	$w_i$	humidity ratio at air-water interface, $kg_{gw}/kg_{d.a}$
$H_{fgw}$	latent heat of vaporization of water of water at $T_w$ , $kJ/kg$		
$h_a$	convective heat transfer coefficient, at air side $kW/m^2 K$		

### *Additional Subscripts*

$s$	saturation
bar	barometric
min	minimum
max	maximum

### *Greek*

$\delta$	enthalpy correction factor
$\varepsilon$	tower effectiveness



- 1 flow meter
  - 2 control valve
  - 3 monometer
  - 4 temperature indicator
  - 5 recirculation
  - 6 thermostat
  - 7 pump
  - 8 main switch
  - 9 0.5kW switch
  - 10 1.0kW switch
  - 11 float valve
  - 12 make up tank
  - 13 0.5 kW heater
  - 14 1.5 kW heater
  - 15 air distribution chamber
  - 16 basin
  - 17 packing
  - 18 damper
  - 19 water distribution system
  - 20 droplet arrester
  - 21 orifice
  - 22 connection for orifice differential pressure
  - 23 pressure
  - 24 connection for orifice pressure drop across packing
  - 25 cap
  - 26 packing column
  - 27 base unit
  - 28 t<sub>6</sub> water outlet temperature
  - 29 t<sub>4,13</sub> air outlet temperatures
  - 30 t<sub>1,12</sub> air inlet temperatures
  - 31 t<sub>3</sub> water inlet temperature
  - 32 fan
  - 33 air inlet
- cross-sectional view of  
 packing coulmn  
 (150mmx150mmx800mm high)  
 number of deck 8  
 packing density 110 or 70 and 200.

Fig. 1 Details of cooling tower test rig



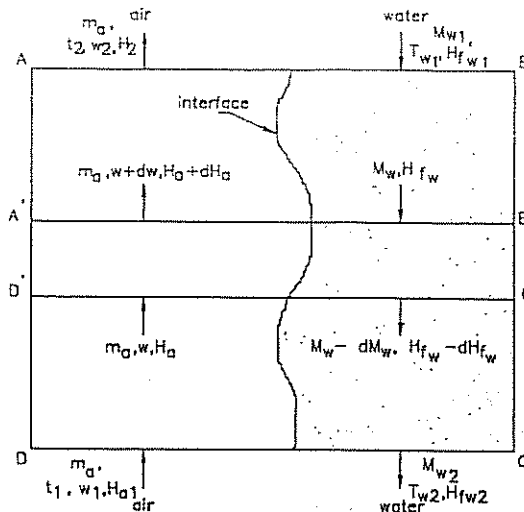


Fig. 2 Schematic diagram of counter flow cooling tower

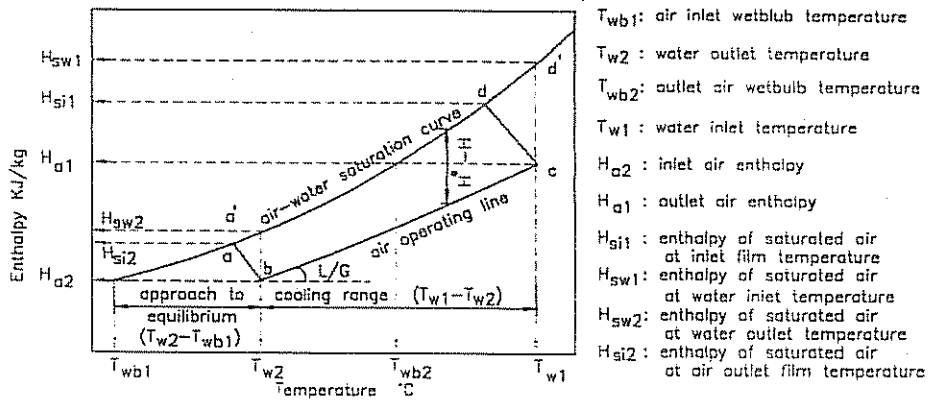


Fig. 3 Thermal performance of cooling tower with definitions of the parameter used

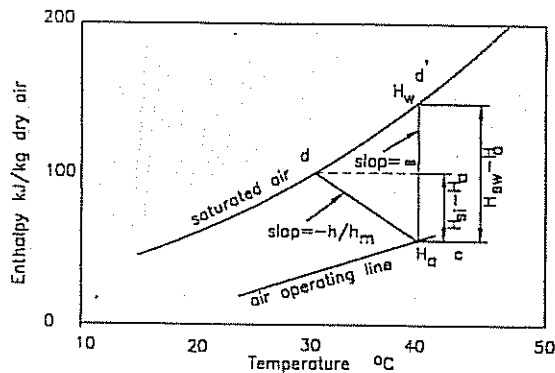


Fig. 4 General operating diagram for cooling tower with true and apparent driving force

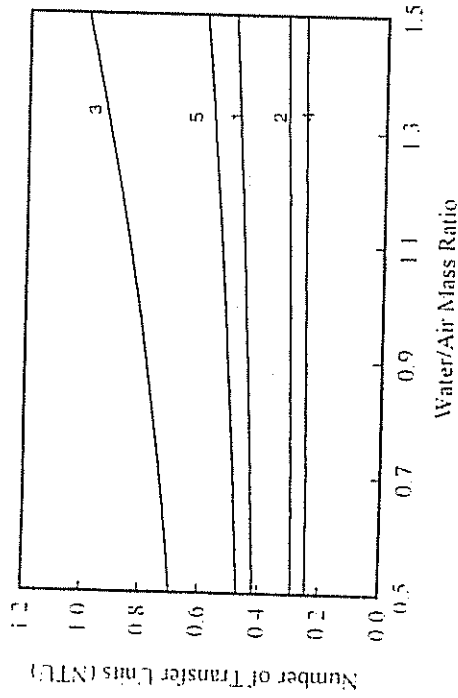


Fig. (5-a) - Number of transfer units (according to Merkel [2])

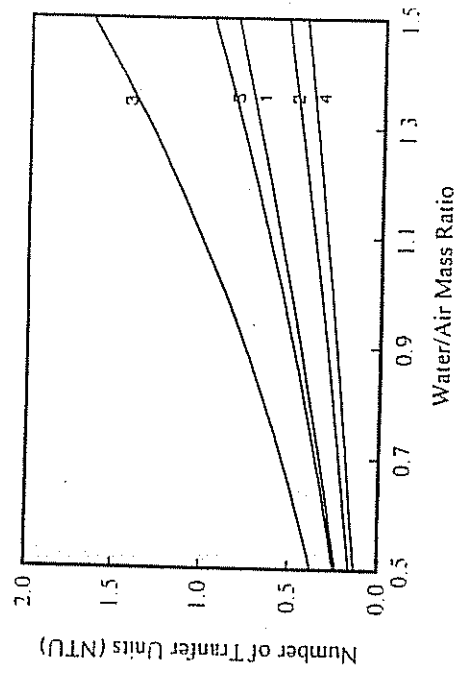


Fig. (5-b) - Number of transfer units taking into account water evaporation

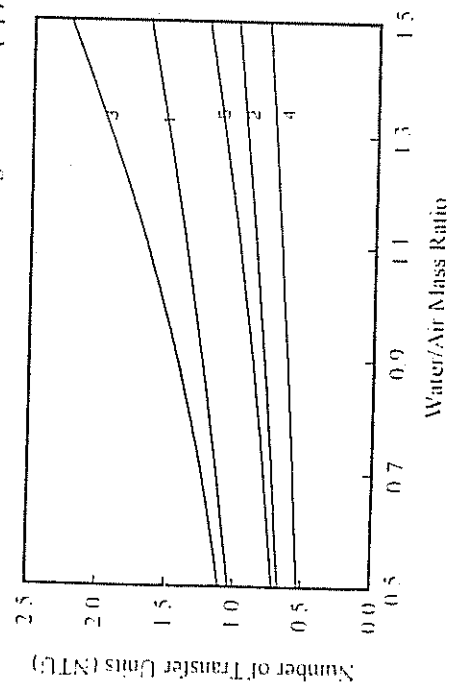


Fig. (5-c) - Number of transfer units taking into account the water evaporation and water/air interface thermal resistance

The number on each curve refers to the run number in Table 2



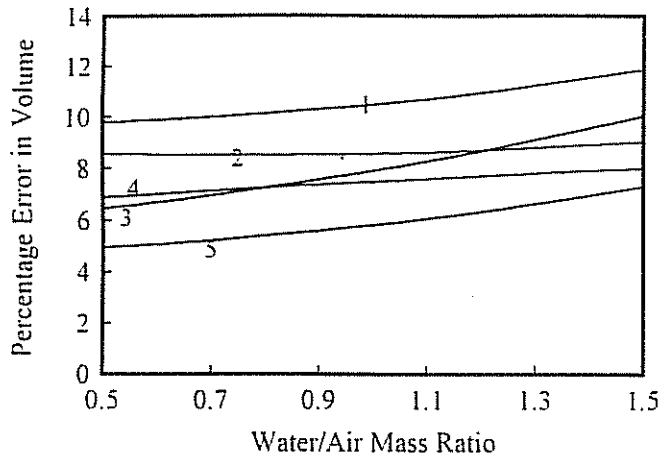


Fig. 6 Variation of percentage error in cooling tower volume with water/air mass ratio due to neglecting the effect of water evaporation in the analysis. The number on each curve refers to the run number in Table 2

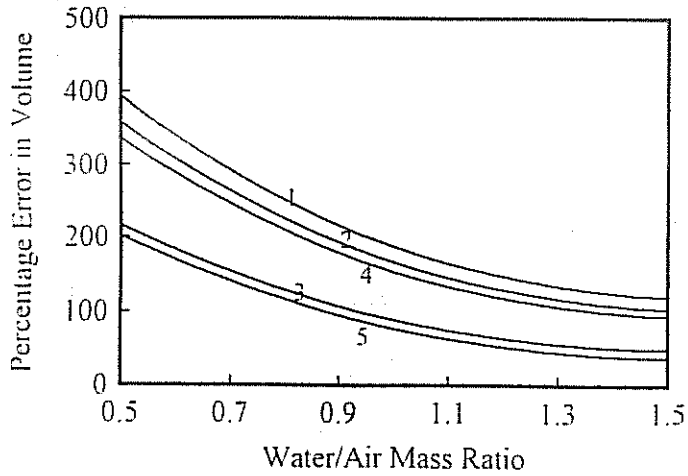


Fig. 7 Variation of percentage error in cooling tower volume with water/air mass ratio due to neglecting the effect of water evaporation and water/air interface thermal resistance in the analysis. The number on each curve refers to the run number in Table 2

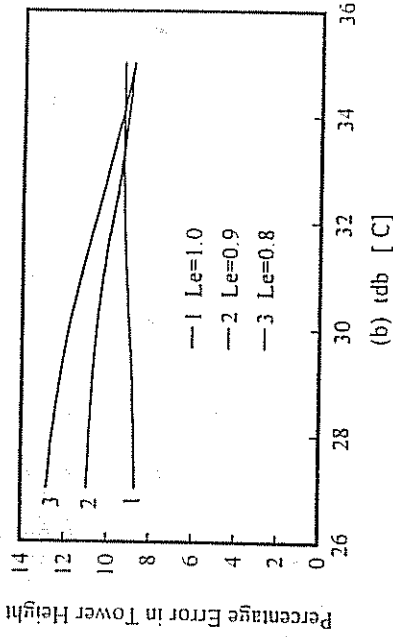
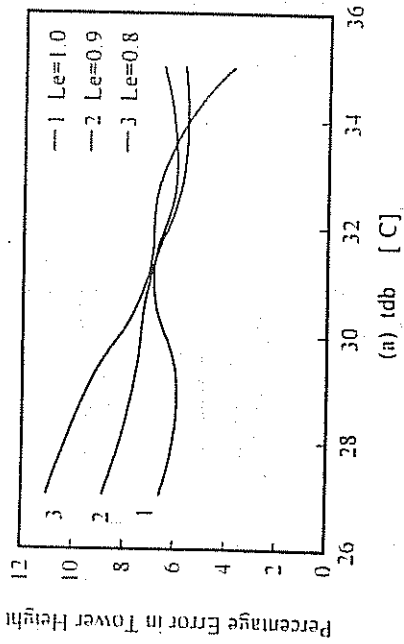


Fig. 8 Effect of air inlet conditions on the percentage error in cooling tower height due to water evaporation for different Lewis number  
 (a)  $T_{wi} = 35^{\circ}\text{C}$ , (b)  $T_{wi} = 40^{\circ}\text{C}$

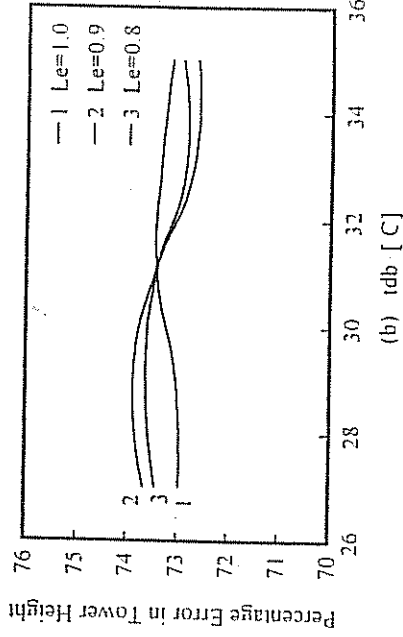
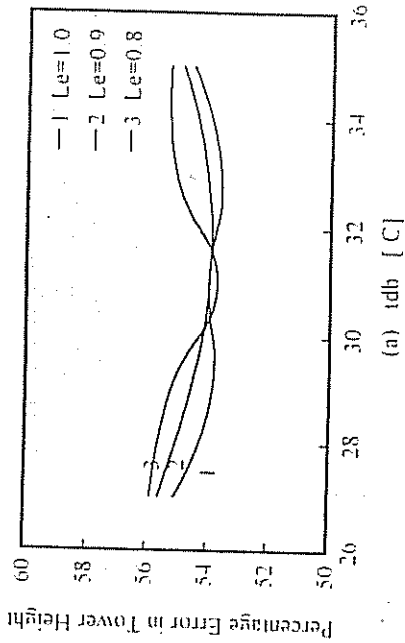


Fig. 9 Effect of air inlet conditions on the percentage error in cooling tower height due to water evaporation and interface film thermal resistance for different Lewis number, (a)  $T_{wi} = 35^{\circ}\text{C}$ , (b)  $T_{wi} = 40^{\circ}\text{C}$

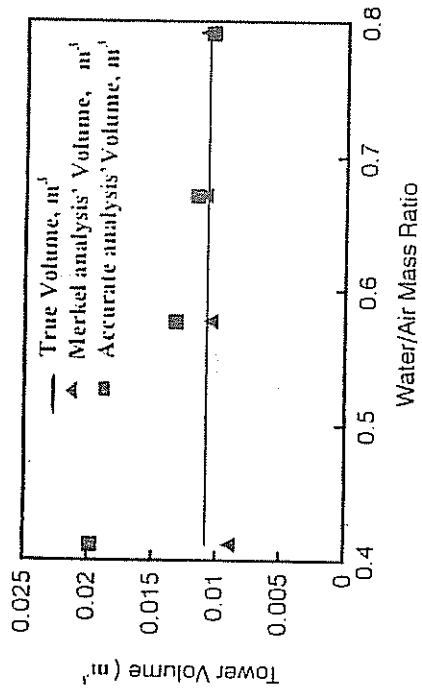


Fig. 10 Comparison between the computed volume of the cooling tower and its true volume

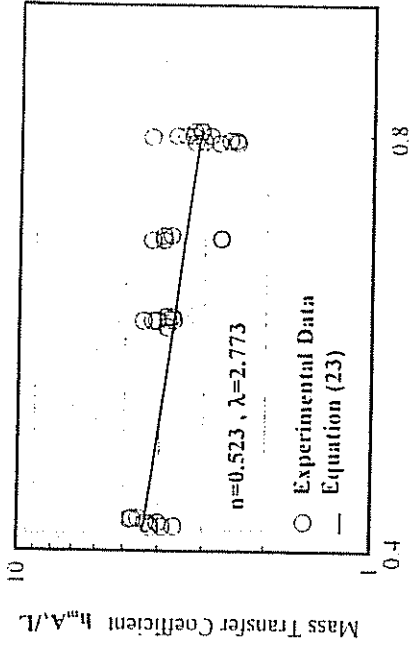


Fig. 11 Variation of mass transfer coefficient with water/air mass flow rate ratio for the present tower

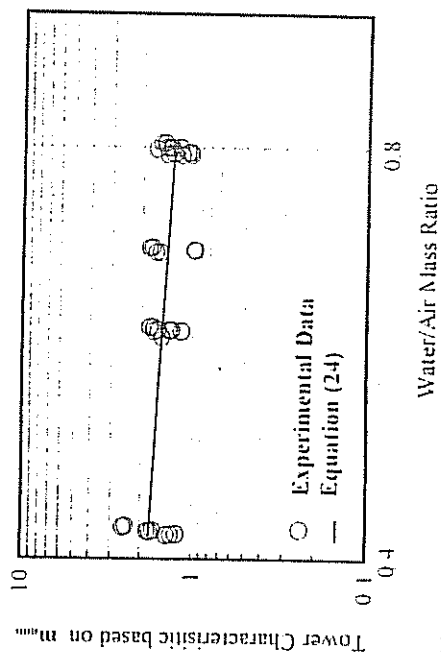


Fig. 12 Variation of the present tower characteristic based on water flow rate with water/air mass ratio

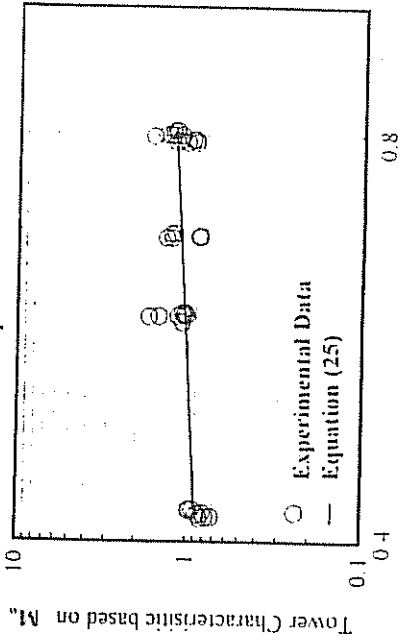


Fig. 13 Variation of the present tower characteristic based on minimum mass flow rate with water/air mass ratio